Hyperparameters optimization for quantum SAT classificator.

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Introduction

In this poster I report numerical results for modeling of 2-MAX-SAT as the problem of the Ising model and using quantum simulator `SimCIM` [1] to solve this model. I show an algorithm of reduction from 2-CNF form into Ising models coefficient and our approach for optimization for `SimCIM` using hyperopt. I used `SimCIM` as classification for two classes $SAT = 1$ $UNSAT = 0$. Optimization has contributed to improved results for 2-MAX-SAT.

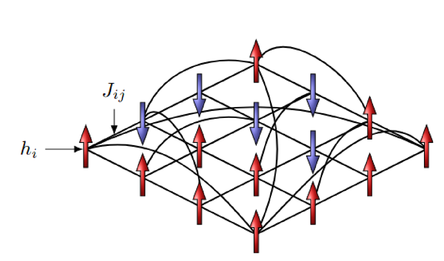
2-SAT and 2-MAX-SAT

The 2-SAT is problem of determining the existence of a solution with two variables in one clause.
An example of a problem instance of 2-SAT and his 2-CNF(Conjuctive Normal Form)

$$F = (x_3 \vee x_2) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2),$$

e.g $\Omega_1 = (x_3 \vee x_2)$ defines clause.
A related optimization problem known as 2-MAX-SAT is determining the maximum number of clauses, of a given Boolean formula in 2-CNF, that can be made true by an assignment of truth values to the variables $\{x_i\}_{i=1}^{N}$ of the formula. The 2-MAX-SAT is an NP-Hard problem.

The Ising Model



The Ising model is the mathematical description of phase transition in magnetic fields.
The model is defined as weighted graph $G = (V, E)$ where V is set of vertex, E is set of edges. Description of energy (Hamiltonian)

$$H(s) = \sum_{\{ij\} \in E} J_{i,j} s_i s_j + \sum_{j \in V} h_j s_j$$

Here, $J_{i,j}$ correspond to weights of the edges and h_o are biases associated between spins s_i, s_j. Typically interested in finding a particular spin configuration $s^* = \arg\min H(s)$. Finding spin configuration is NP-Hard.

2-MAX-SAT into ISING MODEL

2-SAT is solvable in polynomial time.
For each of two variables in clause

$$\Omega_k \text{ where } k \in \{1, 2, \ldots, M\}$$

is defined variable:

$$v_j^k = \begin{cases} -1 & \text{if } x_j \in \Omega_k \text{ and is negation} \\ 1 & \text{if } x_j \in \Omega_k \text{ and is not negation} \\ 0 & \text{if don't occurs in } k\text{th caluse} \end{cases}$$

Base on work from [1] and [2] for formulation I used an algorithm presented below:

procedure $2\text{CNFToIsing}(F)$
- $M \leftarrow$ no. of clauses for formula F
- $N \leftarrow$ no. of variables for formula F
- $h \leftarrow$ create a zero vector size N
- $J \leftarrow$ create zero marix size $N \times N$
- $v \leftarrow$ create zero matrix size $M \times N$
- **for** $j = 1..M$ **do**
  - get index of variables i_1, i_2
  - $var_1, var_2 \leftarrow +1$, -1, 0 as in v_j^k
  - $v[j, i_1] \leftarrow var_1$
  - $v[j, i_2] \leftarrow var_2$
- **for** $j = 1..n$ **do**
  - **for** $i = 1..M$ **do**
    - $j_1 \leftarrow -1$
    - $j_2 \leftarrow 0$
    - **if** $v[j,i] <> 0 \ \& \ j_1 == -1$ **then**
      - $j_1 \leftarrow i + 1$
    - **if** $v[j,i] <> 0 \ \& \ j_1 <> -1$ **then**
      - $j_2 \leftarrow i + 1$
      - **break**
  - $J[j_1, j_2] \leftarrow J[j_1, j_2] + v[j, j_1] \cdot v[j, j_2]$
  - $h[j_1] \leftarrow h[j_1] - v[j, j_1]$
  - $h[j_2] \leftarrow h[j_2] - v[j, j_2]$
- **return** J, h

I obtain the local fields h and coupling J in terms of the parameters of the given 2-SAT instance. Energy of Ising is expressed as:

$$E_{Prob} = \frac{1}{4} E_{Ising} - \frac{1}{4} M \qquad (1)$$

If $E_{Prob} = 0$ then F is satisfiability $SAT = 1$.

The coherent Ising machine

The coherent Ising machine (CIM) [2] [a] enables efficient sampling of low-lying energy states of the Ising Hamiltonian. Hyperparameters of `SimCIM`:

parametr	oznaczenie dla `SimCIM`
N	number of iterations
sigma	noise level
dt	learning rate
zeta	coupling(zeta)
attempt_num	ample size
alpha:	parametr of method SGD [b]

Optimization for hyper-parameters `SimCIM`

I am are interested in random formulas 2-CNF generated by choosing uniformly at random (with fixed clause density, form amlong all possible clauses. The clause density is defined as:

$$\alpha = \frac{M}{N} \qquad (2)$$

where $M =$ no. of clauses and $N =$ no. of variables.
I defined a priors distributions:

parameter	a priori
N	quniform$(300, 500, 10)$
sigma	uniform$(0.0, 1.0)$
dt	loguniform$(\ln(0.01), \ln(0.01))$
zeta	uniform$(0.0, 2.0)$
attempt_num	quniform$(950, 1100, 5)$
alpha	uniform$(0.1, 0.9)$

The objective function is defined as:

$$c(y, \hat{y}) = \frac{1}{N} \sum_{i=0}^{N} \mathbb{1}(y_i \neq \hat{y})$$

I using generator from work: [3], adapted for this work [c] My results for optimization is here: github.com/marcin119a/sat-experiments

[a] This simulator was developed by prof. A. I. Lvovsky.
[b] Stochastic gradient descent
[c] `https://bit.ly/342HFKq`

Model evaluation

The graphs show a comparison of the simulator's operation for $N = 50$ and $N = 100$ variables per formula.

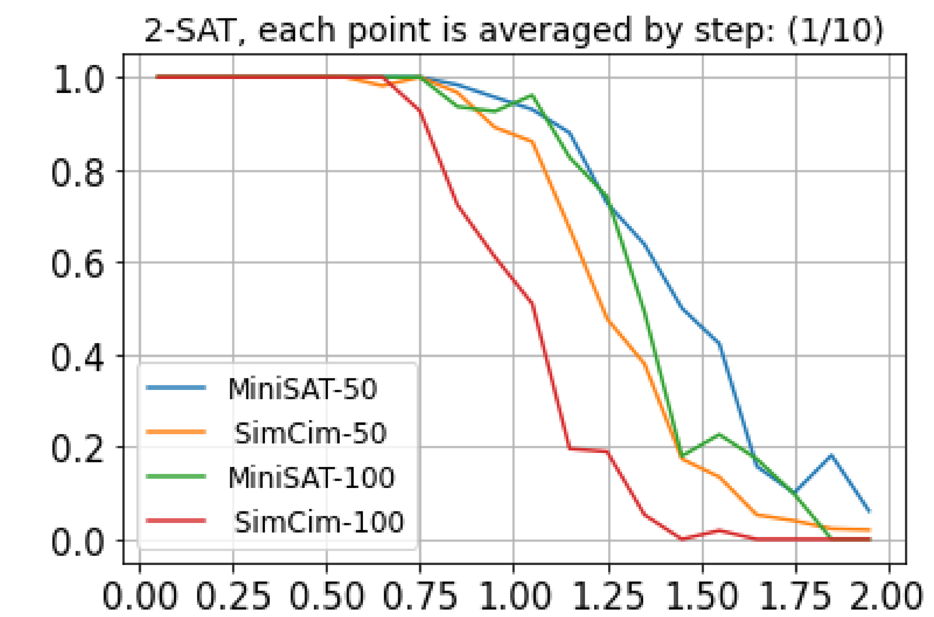


Figure: Fraction of satisfiability formulas depends of picking α

Future work

- Calculate a probability sampling low energy state without finding a ground state.
- Investigate a phase transition without finding an exact ground state.
- Provide new metrics for loss, defined as subset maximum satisfiable percent, depending on critical point $= 1$

REFERENCES

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[2] Egor S. Tiunov, Alexander E. Ulanov, and A. I. Lvovsky.
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